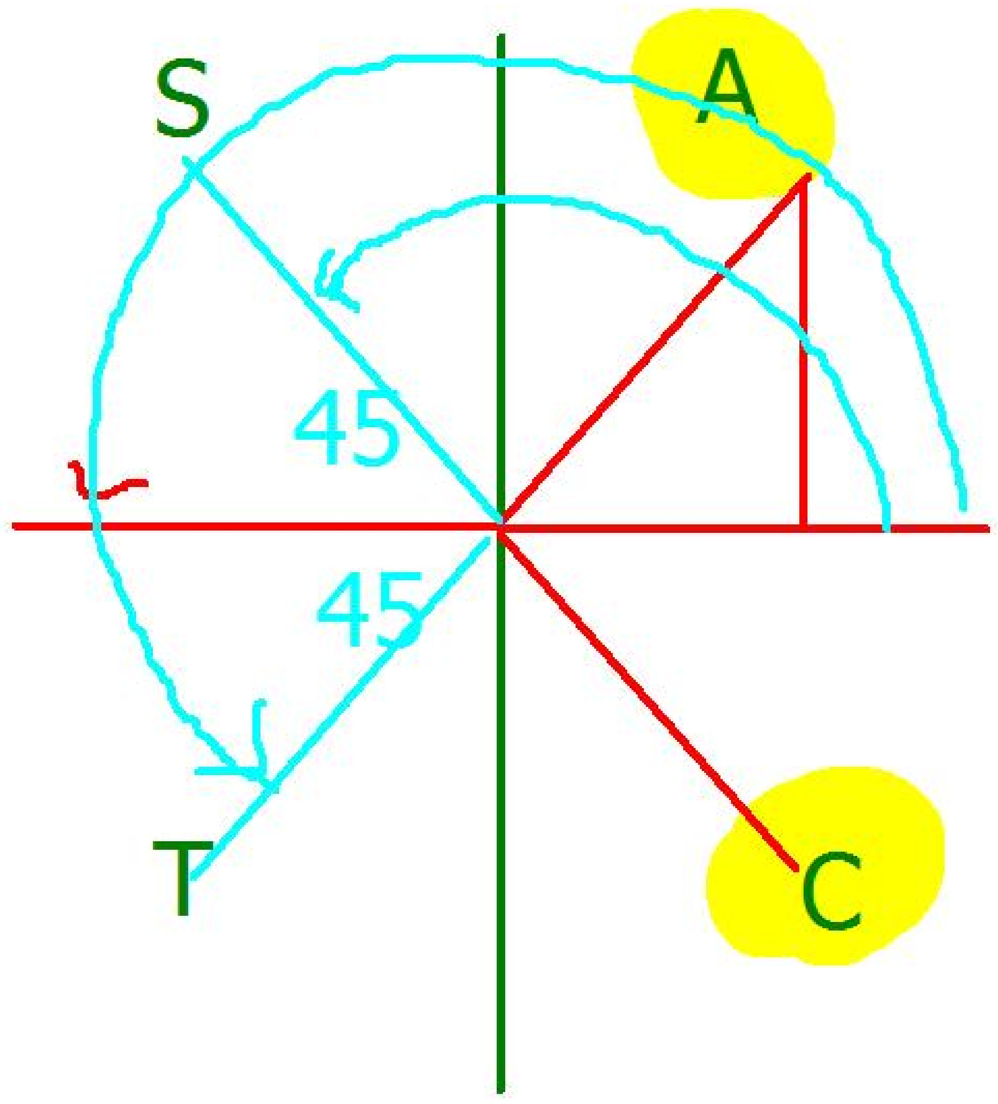
14.4 Solve Trigonometric Equations

Goal • Solve trigonometric equations.

Your Notes



Example 1 ***Solve a trigonometric equation in an interval***

Solve $2\cos^2 x + 1 = 2$ in the interval $0 \le x \le 3\pi$.

Solution

$2\cos^2 x + 1 = 2$ — **Write original equation.**

$2\cos^2 x = \underline{1}$ — **Subtract 1 from each side.**

$\cos^2 x = \underline{\frac{1}{2}}$ — **Divide each side by 2.**

$\cos x = \underline{\pm\frac{\sqrt{2}}{2}}$ — **Take square roots of each side.**

$x = \cos^{-1}\frac{\sqrt{2}}{2}$ or $x = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$x = \frac{\pi}{4}$ or $x = -\frac{\pi}{4}$ $\qquad$ $x = \frac{3\pi}{4}$ or $x = \frac{5\pi}{4}$

Therefore, the general solution of the equation is:

$x = \frac{\pi}{4} + \underline{2\pi n}$ or $x = -\frac{\pi}{4} + \underline{2\pi n}$ or

$x = \frac{3\pi}{4} + \underline{2\pi n}$ or $x = \frac{5\pi}{4} + \underline{2\pi n}$

where n is any integer.

To write the general solution of a trigonometric equation, you can add multiples of the period to all the solutions from one cycle.

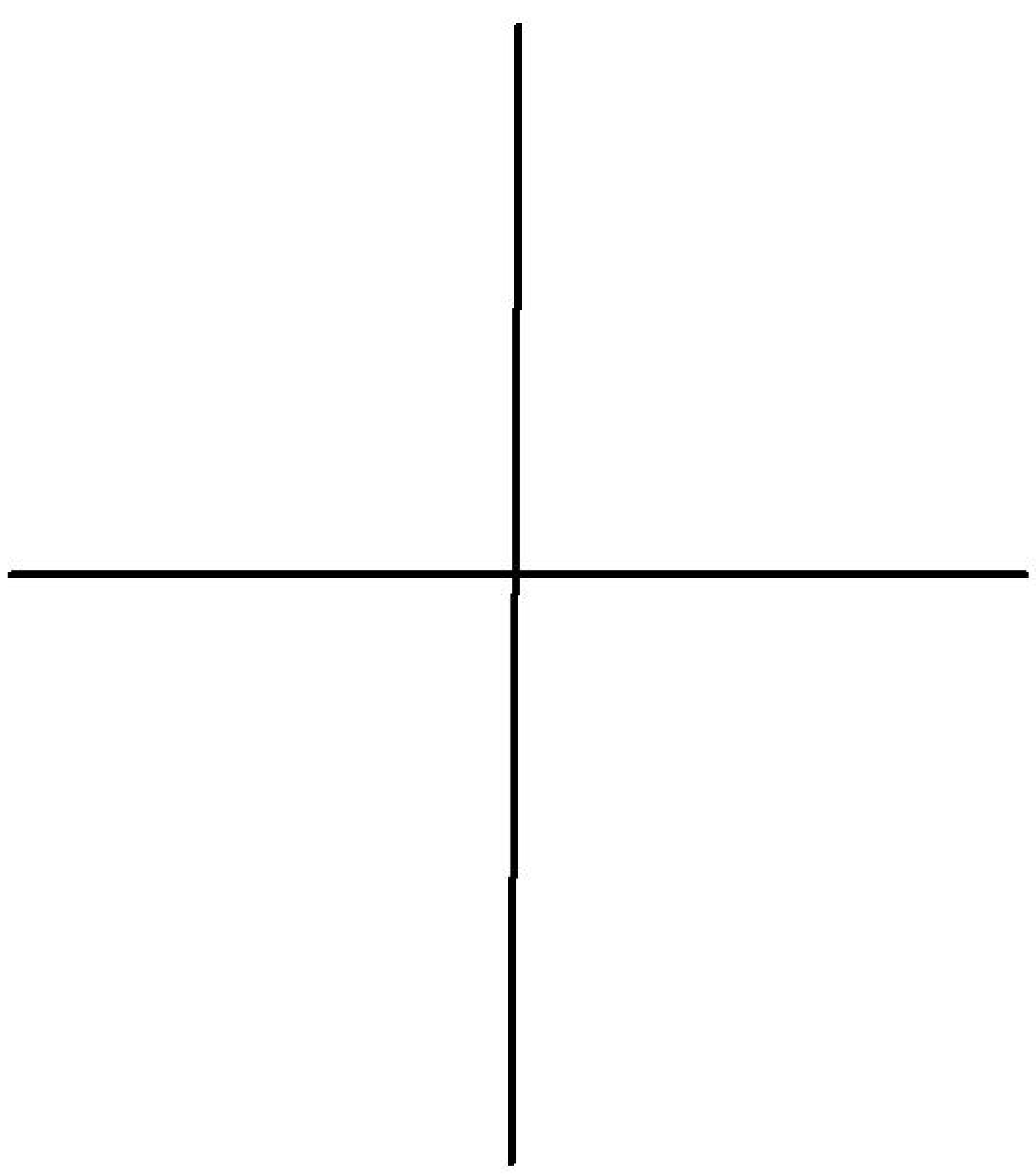
The specific solutions that are in the interval $0 \le x \le 3\pi$ are:

$x = \frac{\pi}{4}$ $\qquad$ $x = \frac{3\pi}{4}$

$x = \frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$ $\qquad$ $x = \frac{3\pi}{4} + 2\pi = \frac{11\pi}{4}$

$x = -\frac{\pi}{4} + 2\pi = \frac{7\pi}{4}$ $\qquad$ $x = \frac{5\pi}{4}$

Your Notes



Example 2 ***Solve a trigonometric equation in an interval***

Solve $d = 20 - 12 \sin \frac{\pi t}{4}$ in the interval $0 \le t \le 24$ when $d = 8$.

Solution

$20 - 12 \sin \frac{\pi t}{4} = 8$ — **Substitute 8 for d.**

$-12 \sin \frac{\pi t}{4} = -12$ — **Subtract 20 from each side.**

$\sin \frac{\pi t}{4} = 1$ — **Divide each side by -12.**

$\frac{\cancel{\pi} t}{4} = \frac{\cancel{\pi}}{2} + 2\cancel{p}in$ — **$\sin \theta = 1$ when $\theta = \frac{\pi}{2} +$ 2pin.** 2n*pi

$t =$ 2+8n ✓ — **Solve for t.** n=0,1,2,3,4,5.......

On the interval $0 \le t \le 24$, d is 8 when
$t =$ 2+8(0) $=$ 2, $t =$ 2+8(1) $=$ 10, and
$t =$ 2+8(3) $=$ 18. ~~2+8(4) =34~~

Example 3 ***Use the quadratic formula***

Solve $2 \sin^2 x + 5 \sin x + 3 = 0$ in the interval $-\pi \le x \le \pi$.

$$\boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Solution

$2 \sin^2 x + 5 \sin x + 3 = 0$ — **Write original equation.**

$2a^2+5a+3 = 0$

$\sin x = \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)}$ — **Quadratic formula**

$= \frac{-5 \pm \sqrt{1}}{4}$ — **Simplify.**

$= -1$ ✓ or $-\frac{3}{2}$ ✓ — **Simplify.**

$x = \sin^{-1}$ -1 or $x = \sin^{-1}$ -1.5 — **Use inverse sine.**

$=$ -90   No Solution — $\boxed{\textbf{Use a calculator.}}$

In the interval $\boxed{-\pi \le x \le \pi}$, the only solution is $x =$ -90.

Your Notes

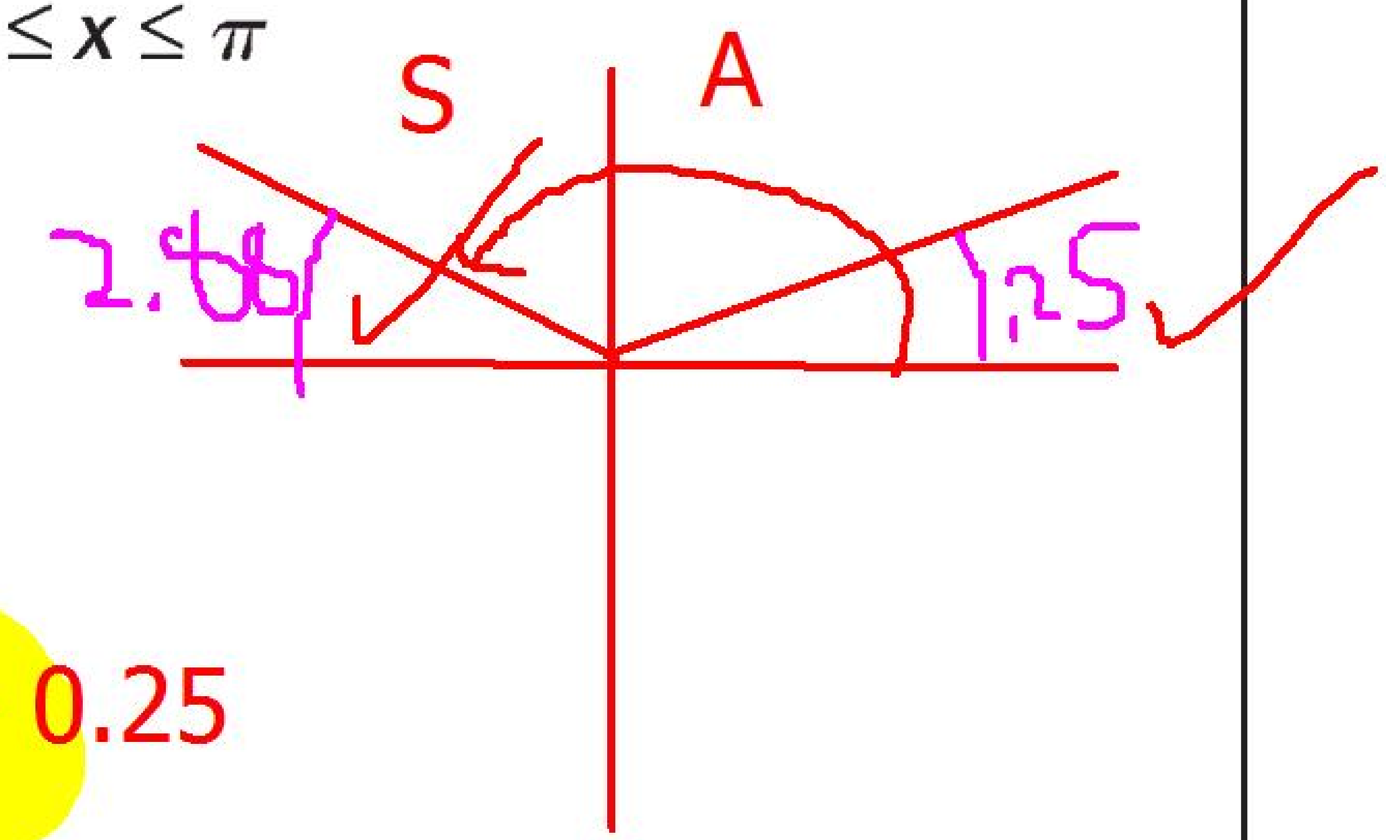
✔ *Checkpoint* Solve the trigonometric equation in the interval.

1. $16 \sin^2 x + 5 = 6;\ 0 \le x \le \pi$

$16\sin^2 x = 1$

$\sin^2 x = \dfrac{1}{16}$

$\sin x = \sqrt{\dfrac{1}{16}} = \pm 0.25$



√ **2.** $20 - 12 \sin \frac{\pi t}{4} = 25;\ 0 < t < 3\pi$

√ **3.** $\cos^2 x + 3 \cos x - 4 = 0;\ 0 \le x \le \pi$

Your Notes

Example 4 *Solve an equation with an extraneous solution*

Solve $1 - \cos x = \sqrt{3} \sin x$ in the interval $0 \le x < \pi$.

Solution

$$1 - \cos x = \sqrt{3} \sin x$$

$$(1 - \cos x)^2 = \_\_\_\_\_\_$$

$$1 - 2 \cos x + \cos^2 x = \_\_\_\_\_\_$$

$$1 - 2 \cos x + \cos^2 x = 3\_\_\_\_\_\_$$

$$1 - 2 \cos x + \cos^2 x = \_\_\_ - \_\_\_\_\_\_$$

$\_\_\_\_\_\_\_\_\_\_ = 0$ **Quadratic form**

$\_\_\_\_\_\_\_\_\_\_ = 0$ **Divide each side by 2.**

$\_\_\_\_\_\_\_\_\_\_ = 0$ **Factor.**

$\_\_\_\_\_\_\_\_$ or $\_\_\_\_\_\_\_\_$ **Zero product property**

$\cos x = \_\_\_$ or $\cos x = \_\_\_$ **Solve for cos *x*.**

$x = \_\_\_$ or $x = \_\_\_$ $x = \_\_\_$ **Solve for *x*.**

The apparent solution ___ does not check in the original equation. The only solutions in the interval $0 \le x \le 2\pi$ are $x = \_\_\_$ and $x = \_\_\_$.

Checkpoint **Complete the following exercise.**

4. Solve the equation in Example 4 in the interval $0 \le x < 4\pi$.

Homework